Principles to Actions
Executive Summary

In 1989 the National Council of Teachers of Mathematics (NCTM) launched the standards-based education movement in North America with the release of *Curriculum and Evaluation Standards for School Mathematics*, an unprecedented initiative to promote systemic improvement in mathematics education. Now, twenty-five years later, the widespread adoption of college- and career-readiness standards, including adoption in the United States of the Common Core State Standards for Mathematics (CCSSM) by forty-five of the fifty states, provides an opportunity to reenergize and focus our commitment to significant improvement in mathematics education.

What is *different* and *promising* today, however, is the hope that the implementation of common standards, and the new generation of aligned and rigorous assessments, will help to address the continuing challenges and expand the progress already made. The need for coherent standards that promote college and career readiness has been endorsed across all states and provinces, whether or not they have adopted CCSSM. As NCTM publicly declared in the Position Statement *Supporting the Common Core State Standards for Mathematics*, released in 2013,

> The widespread adoption of the Common Core State Standards for Mathematics presents an unprecedented opportunity for systemic improvement in mathematics education in the United States. The Common Core State Standards offer a foundation for the development of more rigorous, focused, and coherent mathematics curricula, instruction, and assessments that promote conceptual understanding and reasoning as well as skill fluency. This foundation will help to ensure that all students are ready for college and the workplace when they graduate from high school and that they are prepared to take their place as productive, full participants in society.

What is the *same* today is the need to understand that standards do not teach; teachers teach. New standards provide guidance and direction, and help focus and clarify common outcomes. They motivate the development of new instructional resources and assessments. But these standards do not tell teachers, coaches, administrators, parents, or policymakers what to do at the classroom, school, or district level or how to begin making essential changes to implement the standards. Moreover, they do not describe or prescribe the essential conditions required to ensure mathematical success for all students. Thus, the primary purpose of *Principles to Actions* is to fill this gap between the development and adoption of CCSSM and other standards and the enactment of practices, policies, programs, and actions required for their widespread and successful implementation. Its overarching message is that effective teaching is the nonnegotiable core that ensures that all students learn mathematics at high levels and that such teaching requires a range of actions at the state or provincial, district, school, and classroom levels.

Looking back at mathematics education and student achievement in mathematics, we find much to celebrate. As a result of the gradual implementation of a growing body of research on teaching and learning mathematics and the dedicated efforts of nearly two million teachers of mathematics in North America, student achievement is at historic highs. For example, the percentage of fourth graders scoring "proficient" or above on the National Assessment of Educational Progress (NAEP) rose from 13 percent in 1990 to 42 percent in 2013, and the percentage of eighth graders scoring "proficient" or above on the NAEP rose from 15 percent in 1990 to 36 percent in 2013. Between 1990

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and 2013, mean SAT-Math scores increased from 501 in 1990 to 514 in 2013, mean ACT scores increased from 19.9 to 20.9, and the number of students taking Advanced Placement examinations in calculus and statistics increased substantially, from 77,634 in 1982 to 387,297 in 2013, and from 7,667 in 1997 to 169,508 in 2013, respectively.

These are impressive accomplishments. However, while we celebrate these record high NAEP scores and increases in SAT and ACT achievement—despite a significantly larger and more diverse range of test-takers—other recent data demonstrate that we are far from where we need to be and that much remains to be accomplished. For example, the average mathematics NAEP scores for 17-year-olds has been essentially flat since 1973; the difference in average NAEP mathematics scores between white and black and white and Hispanic 9- and 13-year-olds has narrowed somewhat between 1973 and 2012 but remains between 17 and 28 points; and among cohorts of 15-year-olds from the 34 countries participating in the 2012 Programme for International Student Assessment (PISA), the U.S. cohort ranked 26th in mathematics.

These more disturbing data point to the persistent challenges and the work that we still need to do to make mathematics achievement a reality for all students:

- Eliminate persistent racial, ethnic, and income achievement gaps so that all students have opportunities and supports to achieve high levels of mathematics learning
- Increase the level of mathematics learning of all students, so that they are college and career ready when they graduate from high school
- Increase the number of high school graduates, especially those from traditionally underrepresented groups, who are interested in, and prepared for, STEM careers

In short, we must move from "pockets of excellence" to "systemic excellence" by providing mathematics education that supports the learning of all students at the highest possible level.

To achieve this goal, we must change a range of troubling and unproductive realities that exist in too many classrooms, schools, and districts. *Principles to Actions* addresses and documents these realities:

- Too much focus is on learning procedures without any connection to meaning, understanding, or the applications that require these procedures.
- Too many students are limited by the lower expectations and narrower curricula of remedial tracks from which few ever emerge.
- Too many teachers have limited access to the instructional materials, tools, and technology that they need.
- Too much weight is placed on results from assessments—particularly large-scale, high-stakes assessments—that emphasize skills and fact recall and fail to give sufficient attention to problem solving and reasoning.
- Too many teachers of mathematics remain professionally isolated, without the benefits of collaborative structures and coaching, and with inadequate opportunities for professional development related to mathematics teaching and learning.

As a result, too few students—especially those from traditionally underrepresented groups—are attaining high levels of mathematics learning.

In this exciting and challenging context, NCTM introduces *Principles to Actions: Ensuring Mathematical Success for All,* setting forth a set of strongly recommended, research-informed actions, based on the Council's core principles and intended for all educational leaders and policymakers, all school and district administrators, and all teachers, coaches, and specialists of mathematics. In *Principles and Standards for School Mathematics*, published by NCTM in 2000,

the Council first defined a set of Principles that "describe features of high-quality mathematics education." *Principles to Actions* now articulates and builds on an updated set of six Guiding Principles that reflect more than a decade of experience and new research evidence about excellent mathematics programs, as well as significant obstacles and unproductive beliefs that continue to compromise progress.

Three aspects of *Principles to Actions* are new, provocative, and important. First, *Principles to Actions* devotes the largest section to Teaching and Learning, the first Guiding Principle, and describes and illustrates eight Mathematics Teaching Practices (see fig. 1) that research indicates need to be consistent components of every mathematics lesson. Second, for each Guiding Principle, *Principles to Actions* offers commentary and a table that address productive and unproductive beliefs as part of a realistic appraisal of the obstacles that we face, as well as suggestions for overcoming these obstacles. Third, *Principles to Actions* issues a forceful call to action, asserting that all of us who are stakeholders have a role to play and important actions to take if we are finally to recognize our critical need for a world where the mathematics education of our students draws from research, is informed by common sense and good judgment, and is driven by a nonnegotiable belief that we must develop mathematical understanding and self-confidence in *all* students.

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Fig. 1. Mathematics Teaching Practices

Guiding Principles for School Mathematics

Full statements of the Guiding Principles follow; *Principles to Actions* elaborates the unique importance of each, as summarized briefly below each statement. The first Guiding Principle, Teaching and Learning, has primacy among the Guiding Principles, with the others serving as the Essential Elements that support it.

Teaching and Learning. *An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically.*

The teaching of mathematics is complex. It requires teachers to have a deep understanding of the mathematical content that they are expected to teach and a clear view of how student learning of that mathematics develops and progresses across grades. It also calls for teachers to be skilled at using instructional practices that are effective in developing mathematics learning for all students. The eight Mathematics Teaching Practices (see fig. 1) describe the essential teaching skills derived from the research-based learning principles, as well as other knowledge of mathematics teaching that has emerged over the last two decades.

Access and Equity. *An excellent mathematics program requires that all students have access to a high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential.*

Equitable access means high expectations, adequate time, consistent opportunities to learn, and strong support that enable students to be mathematically successful. Instead of one-size-fits-all practices and the differential expectations for students who are placed in different academic tracks, equitable access means accommodating differences to meet a common goal of high levels of learning by all students.

Curriculum. *An excellent mathematics program includes a curriculum that develops important mathematics along coherent learning progressions and develops connections among areas of mathematical study and between mathematics and the real world.*

A robust curriculum is more than a collection of activities; instead, it is a coherent sequencing of core mathematical ideas that are well articulated across the grades. Such an effective curriculum incorporates problems in contexts from everyday life and other subjects whenever possible. These tasks engage students and generate interest and curiosity in the topics under investigation.

Tools and Technology. *An excellent mathematics program integrates the use of mathematical tools and technology as essential resources to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking.*

Available tools and technology help teachers and students visualize and concretize mathematics abstractions, and when these resources are used appropriately, they support effective teaching and meaningful learning.

Assessment. *An excellent mathematics program ensures that assessment is an integral part of instruction, provides evidence of proficiency with important mathematics content and practices, includes a variety of strategies and data sources, and informs feedback to students, instructional decisions, and program improvement.*

Effective assessment supports and enhances the learning of important mathematics by furnishing useful formative and summative information to both teachers and students. Productive mathematics assessment is a process that is

coherently aligned with learning goals and makes deliberate use of the data gathered as evidence of learning and provides guidance for next instructional steps and programmatic decision making. Students learn to assess and recognize high quality in their own work.

Professionalism. *In an excellent mathematics program, educators hold themselves and their colleagues accountable for the mathematical success of every student and for personal and collective professional growth toward effective teaching and learning of mathematics.*

Effective schools communicate a tangible sense of the professional imperative to grow personally and collectively and to hold one another accountable for this growth. Professionals who are responsible for students' mathematics learning are never satisfied with their accomplishments and are always working to increase the impact that they have on their students' mathematics learning. Moreover, they cultivate and support a culture of professional collaboration and continual improvement that is driven by an abiding sense of interdependence and collective responsibility.

Actions

Although principles provide guidance and structure, actions determine impact. *Principles to Actions* argues that ensuring mathematical success for all will take **teachers** who, among other actions—

- plan and implement effective instruction as described by the Mathematics Teaching Practices;
- develop socially, emotionally, and academically safe environments for mathematics teaching and learning—environments in which students feel secure and confident in engaging with one another and with teachers;
- evaluate curricular materials and resources to determine the extent to which these materials align with the standards, ensure coherent development of topics within and across grades, promote the mathematical practices, and support effective instruction that implements the Mathematics Teaching Practices;
- incorporate mathematical tools and technology as an everyday part of the mathematics classroom, recognizing that students should experience "mathematical action technologies" and physical or virtual manipulatives to explore important mathematics;
- provide students with descriptive, accurate, and timely feedback on assessments, including strengths, weaknesses, and next steps for progress toward the learning targets;
- work collaboratively with colleagues to plan instruction, solve common challenges, and provide mutual support as they take collective responsibility for student learning.

Principles to Actions argues that ensuring mathematical success for all will take **principals, coaches, specialists, and other school leaders** who, among other actions—

- make the eight Mathematics Teaching Practices a schoolwide focus that is expected for all teachers to strengthen learning and teaching for all students, and provide professional development, training, and coaching to make the implementation of these practices a priority;
- maintain a schoolwide culture with high expectations and a growth mindset;

- allocate time for teachers to collaborate in professional learning communities;
- support improvement with multifaceted assessments used to monitor progress and inform changes to instruction;
- make the mathematical success of every student a nonnegotiable priority.

Principles to Actions argues that ensuring mathematical success for all will take **leaders and policymakers in districts, states or provinces, including commissioners, superintendents and other central office administrators**, who, among other actions—

- make ongoing professional development that supports the implementation of the eight Mathematics Teaching Practices as a priority;
- allocate resources to ensure that all students are provided with an appropriate amount of instructional time to maximize their learning potential;
- eliminate the tracking of low-achieving students and instead structure interventions that provide high-quality instruction and other classroom support, such as math coaches and specialists;
- understand the devastating impact of professional isolation and create collaborative structures to maximize professional growth;
- Support risk taking and encourage new approaches that advance student learning.

Only when these words become actions and the actions lead to more productive beliefs, new norms of instructional practice, and implementation of the essential supporting elements will we overcome the obstacles that currently prevent school mathematics from ensuring success for all students.

The National Council of Teachers of Mathematics is the world's largest professional organization dedicated to improving mathematics education for all students. Growing out of its visionary *Agenda for Action* in 1980, the Council launched the education standards movement with its publication of *Curriculum and Evaluation Standards for School Mathematics* (1989), which presented a comprehensive vision for mathematics teaching and learning in K–12 mathematics. In 2000, NCTM's *Principles and Standards for School Mathematics* expanded on the 1989 Standards and added underlying Principles for excellence in school mathematics. Subsequent publications, *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* and *Focus in High School Mathematics: Reasoning and Sense Making*, extended this work by identifying the most significant mathematical concepts and skills at each level from prekindergarten through grade 8 and advocating practical changes to the high school mathematics curriculum to refocus learning on reasoning and sense making, respectively. These NCTM publications have significantly influenced the development of mathematics education standards worldwide. NCTM's recently published *Principles to Actions: Ensuring Mathematical Success for All* describes the principles and actions, including specific research-informed teaching practices, that are essential for a high-quality mathematics education for all students. The Council is committed to a constructive public dialogue to ensure a mathematics education of the highest quality for all students.